Mathematics in the Classroom: Conceptual Cartography of Differential Calculus

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Mathematics in the Classroom: Conceptual Cartography of Differential Calculus

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Abstract
This paper presents the results of a documentary investigation with the intention of substantiate how and why, and the level and depth of the topics used by the teacher in the classroom for the development of the mathematical knowledge on the part of higher level engineering students. The analysis of the mathematical object was made through the construction of conceptual cartography, being the core of the derivative concept. To construct the axes, the socio-formative theory of Sergio Tobón was used, together with the semiotic representation register of Raychmond Duval and Tall's mathematical advanced thought in the engineering context. The topic is a part of the Unit of learning: Differential and Integral Calculus. This corresponds to the first semester. The course lasts for a semester and is intended for students aged between 18 and 20 years. The research shows that by constructing a conceptual cartography involving at least 8 axes of analysis that the socio-formation orientates, and taking mathematics in the context of careers offered by the educational institution, the teacher is allowed to place the thematic content in the appropriate level and depth, guiding in a possible treatment of knowledge to be brought into the classroom.

Keywords: conceptual cartography; advanced mathematical thought; socio-formation and mathematics.

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Introduction

In today's society there is no dispute about the importance and relevance of learning mathematics. Today, the challenges of universities, in a knowledge society, require the engineers to be produced in a higher education context in order to be competitive at both the national and international level. Consequently, it is necessary to rethink why mathematics, its content and the teaching methodology, so that students have the ability to be creative, innovative and to solve problems with regard to the area of development that concerns them (García, 2013).

The engineer is likewise identified as a problem solver, an active agent whose raison d’être is to propose and implement solutions to specific problems with the application of the existing knowledge (Hermosillo, 2003).

In addition, Hermosillo goes on to state the development of engineering and mathematics in the Twentieth Century, make it impossible to claim that in a period of four or five years, students know and master all branches of mathematics, and also learn the specifics of their profession. As a result it is necessary to determine which branch of this science is relevant, to what deep and how formally, and which applications are relevant and pertinent to the engineering profession.

One of the problems to be solved is: what is the perspective and depth, and the level of conceptualization of differential calculus that an engineering student must learn so that he can apply this knowledge in any academic, professional or personal context?

The objective of this research report is to foster the understanding of the learning process and some of its elements, so that the teacher can consider and likewise help in an appropriate and relevant way for mathematical engineering education.

The research was qualitative and was based on a conceptual cartography of differential calculus, as part of the course: Differential and Integral Calculus, taught in the first semester of the Escuela Superior de Ingenieria Mecanica y Electrica (ESIME) Unidad Culhuacan of the Instituto Politecnico Nacional (IPN). The students taking the course are aged between 18 to 20 years. The background in relation to the calculus is normally algorithmic.
2. Framework

2.1 As a point of reference, the Mission and Vision of the Instituto Politecnico Nacional (IPN, 2015), were taken. This can be summarized as follows: the Instituto Politecnico Nacional develops professionals who can contribute to the social and technological development of the country, providing educational inputs to all social sectors, putting (as stated in the Institutional slogan) “technics in the service to the Fatherland”.

2.2 The Institutional Educational Model (IPN, 2004), one of the mandatory references, indicates that the student is the center of learning. It also promotes values, skills and knowledge that constitute him as a professional, in order to efficiently solve social, cultural and technological problems related to his work and personal interests. The aim is to promote continuous performance improvement together with social commitment.

2.3 Exit profile. Among the competences to be considered for the exit profile are mainly the problems solutions based in mathematics, the natural sciences and collaborative work (Rodríguez, 2013).

2.4 Socio-formative Focus. From this perspective, the competences are integration of performance and targeted actions to identify, interpret, argue and solve contextual problems, with ethical commitment and continuous improvement, integrating knowledge systematically (Tobon, 2013a)

3. Methodology

The study was conducted by a documentary analysis approach. The analysis and organization of documents such as books, articles and research reports, were carried out based on the elements of conceptual cartography building.

Conceptual cartography is a methodology used for the construction of concepts, based on eight essential dimensions: notional approach, categorization, characterization, differentiation, classes of the concept, entailment, methodology and exemplification (Tobon, 2013b). In conceptual cartography, concepts are mental constructs that allow us to understand the different objects of the inner (subjective) and outer (objective) reality of man, providing classification, characterization, differentiation, composition, attributes and relationships (Tobon, 2004). The analysis axes are followed by a series of guiding questions, building the central topic of the conceptual cartography of a holographic form. The central issue in this case was the
conceptualization of the derivative in the context of the development of an engineer in the area of Communications and Electronics.

4. Results
The results can be seen in the following table

**Table 1.** Conceptual cartography analysis axis of the derivative concept.

<table>
<thead>
<tr>
<th>Analysis axes</th>
<th>Description axis</th>
<th>Approach from the perspective assumed by the teacher in the context of the engineering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exemplification</td>
<td>It describes the application of the concept using understandable examples for the student</td>
<td>The derivative exemplified by engineering phenomena modelling. For example, the speed as the rate of change, constructed from mathematics</td>
</tr>
<tr>
<td>Notional</td>
<td>Intuitive approximation to the concept from different definitions, relating them to different contexts</td>
<td>It is showed the physical, graphical, mathematical approach. (It induces the managing of different semiotic representations of the mathematical object)</td>
</tr>
<tr>
<td>Categorization</td>
<td>It describes the general class of concepts included inside the topic.</td>
<td>It allows us to place the concept as part of the mathematical analysis (mathematics area). This differentiates its construction from the engineering context.</td>
</tr>
<tr>
<td>Characterization</td>
<td>It describes the essential characteristics of the concept</td>
<td>It allows us to integrate knowledge and its characteristics in the neighbourhood of the central topic.</td>
</tr>
<tr>
<td>Differentiation</td>
<td>It describes the difference with similar concepts.</td>
<td>Distinguishing the concepts induces a better conceptualization that allows the construction of theory based in the central concept.</td>
</tr>
<tr>
<td>Classes of de Concept</td>
<td>It divides the concept or focuses it from diverse perspectives.</td>
<td>It induces strategies to approach the concept. In this case by means of the diverse register of representation.</td>
</tr>
<tr>
<td>Entailment</td>
<td>It establishes the relationship of the central concept with</td>
<td>It lets us identify the educational intention and link it with the strategies in the classroom.</td>
</tr>
</tbody>
</table>
5. Discussion

Through the analysis of the conceptual cartography of differential calculus, one can see mathematics as a helping tool to build the subject curriculum, to solve problems in the field of engineering and to promote a mathematical thinking.

The use of conceptual cartography encourages the teacher to manipulate scientific knowledge and lead to the corresponding academic field of engineering to give the student sense and meaning to what he learns.

In addition, conceptual cartography with regard to differential calculus suggests that mathematics teachers need to have academic teaching knowledge. In this case, this is shown by the linking of the axis with the educational and engineering context. In mathematical education research, results contribute to the promotion of advanced mathematical thinking. This involves cognitive processes such as abstraction that can be understood as replacing concrete phenomena by cognitive processes and concepts that are only confined to the human mind. Abstraction is not typical of advanced mathematical thinking. However, it becomes increasingly important in higher level courses when the students analyse, categorize, conjecture, generalize, synthesize, define, prove and formalize (Azcarate, Camacho, 2003).

Teachers ought to develop teaching and learning strategies to facilitate the construction of knowledge of any student. In the case of mathematics, the results of research in the field of mathematics education, show that the use of registers of semiotic representation contribute to the mathematical learning when they are incorporated into educational activities (Ibarra, Bravo, Grijalva, 2001), since as a characteristic of the specific structures and mathematical concepts, it is needed to use different representations to assimilate the concepts and understand them in all their
complexity, from a cognitive perspective. This is necessary for a complete understanding of mathematical knowledge (Sánchez, 2014); in fact Duval (1993) in (Ibarra et al, pp. 108, 2001) says "The conceptual understanding of mathematical content lies in coordinating at least two registers of representation and this coordination is manifested by the speed and spontaneity of the cognitive conversion activity."

6. Final Considerations

In this documentary study the conceptual cartography of differential calculus (or any other mathematical topic to be taught in the classroom), has been implemented as a strategy for educational research for teachers. This will allow the teacher to guide knowledge in terms of its depth, relevance, meaning and significance, with reference to the complex thinking and socio-formation. This will allow the development of a complete and pertinent education to the knowledge society, and will provide a framework that can be used to build and communicate scientific knowledge at the academic level (Tobon, 2004)

The use of conceptual cartography in terms of socio-formation as educational research strategy has allowed the creation of a holographic view of the central themes, showing relationships and linkages with regard to scientific knowledge, the academic context and areas, such as mathematical education. This in turn will assist in the teaching and continuous improvement in the teaching of mathematics.

References


Biodata

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